ALGEBRAIC IDENTITIES AND FORMULAE

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$

3. $a^2 + b^2 = (a+b)^2 - 2ab$
4. $a^2 + b^2 = (a-b)^2 + 2ab$

5. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$
6. $(a+b)^2 - (a-b)^2 = 4ab$

7. $(a+b)^2 = (a-b)^2 + 4ab$
8. $(a-b)^2 = (a+b)^2 - 4ab$

9. $(a+b)(a-b) = a^2 - b^2$

10. (i) $(x+a)(x+b) = x^2 + (a+b)x + ab$
    (ii) $(x+a)(x-b) = x^2 + (a-b)x - ab$
    (iii) $(x-a)(x+b) = x^2 - (a-b)x - ab$
    (iv) $(x-a)(x-b) = x^2 - (a+b)x + ab$

11. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
12. $a^2 + b^2 + c^2 = (a+b+c)^2 - 2(ab + bc + ca)$

13. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
14. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

15. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
16. $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

17. $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
18. $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$

19. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
20. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

21. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$$

$$= (a + b + c)[(a + b + c)^2 - 3(ab + bc + ca)]$$

22. $If\ (a + b + c) = 0, then\ a^3 + b^3 + c^3 = 3abc$

23. $If\ a^3 + b^3 + c^3 = 3abc, then\ a + b + c = 0\ or\ a = b = c$

24. $If\ a^2 + b^2 + c^2 = ab + bc + ca, then\ a = b = c.$

25. $a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2.$

26. $a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2.$

27. $If\ x + \frac{1}{x} = k,\ then$

$(i)\ x - \dfrac{1}{x} = \sqrt{k^2 - 4}$

$(ii)\ x^2 + \dfrac{1}{x^2} = k^2 - 2$

$(iii)\ x^3 + \dfrac{1}{x^3} = k^3 - 3k$